Nonlinear Model of Superconducting Strip Transmission Lines*

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ABSTRACT

The Ginzburg-Landau theory is used to predict the nonlinear behavior in superconducting strip transmission lines. A method for calculating the nonlinear inductance and the fractional change in the resonant frequency of a stripline resonator is presented. Comparisons with measurements for two $YBa_2Cu_3O_{7-x}$ stripline resonators show excellent agreement.

INTRODUCTION

This paper summarizes the first rigorous effort to model the nonlinear response in superconducting transmission lines and compare the calculations with experimental data for $YBa_2Cu_3O_{7-x}$. For modeling the nonlinearity in superconductors, we have chosen the Ginzburg-Landau (GL) theory [1, 2] since it is a well known and widely accepted formulism which deals with the nonlinear response to fields and currents strong enough to change the superconducting carrier density. It is the aim of this paper to apply the GL theory to the nonlinear modeling of superconducting strip transmission lines. Accurate modeling of the nonlinear effects is important for device applications. A more detailed description of this work and its results will be published soon [3].

STRIPLINE MODEL

In modeling superconducting strip transmission lines operating in the low gigahertz region, we can assume that the current is flowing in the longitudinal direction, that is, $\bar{J} = \hat{z}J_z$, and the transverse components J_x and J_y can be neglected. We also assume that the time-independent GL theory can be applied. In this case, if the normalized complex order parameter $\psi' = u\exp(i\theta) = \psi/\psi_0$, where ψ is the GL order parameter and ψ_0 is the order parameter at zero applied field, it is straightforward to show that the two-dimensional (2-D) GL equations can be simplified to

$$\nabla_s'^2 A_z' = u^2 A_z' \tag{1}$$

$$\frac{1}{\kappa^2}\nabla_s'^2 u = u\left(u^2 - 1 + \frac{A_z'^2}{2}\right) \tag{2}$$

where,

$$\nabla_s' = \hat{x}\frac{\partial}{\partial x'} + \hat{y}\frac{\partial}{\partial y'}$$

κ is the GL parameter,

$$A_z' = \frac{e^*|\psi_0|}{\sqrt{m^*\mu_0 H_c}} A_z,$$

$$u = |\psi| / |\psi_0| = \lambda_L / \lambda,$$

λ_L is the low-field London penetration depth, H_c is the thermodynamic critical field, and e* and m* are the charge and mass, respectively, of the superconductoring pairs. The boundary conditions for A_z' and u are given by

$$\nabla_s' \times \hat{z}A_z' = \mu_0 \bar{H}_t' \tag{3}$$

$$n \cdot \nabla_s' u = 0 \tag{4}$$

where $\bar{H}_t'$ is the normalized transverse magnetic field $= \bar{H}_t' / H_c$.

Consider a superconducting stripline shown in Figure 1. The linear characteristics of such a stripline have been analyzed in [4]. It was found that the peak current density in the center strip conductor is at least two orders higher in magnitude than that in the ground plane. In view of this, the nonlinearity is considered to be mainly due to the center strip conductor and can be neglected in the ground planes.

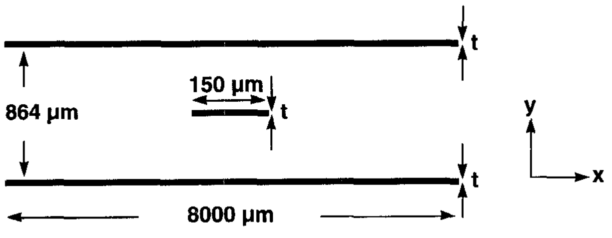

Figure 1. Superconducting strip transmission line.

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In order to obtain the nonlinear characteristics of superconducting striplines, we need to solve the 2-D GL equations for different input currents. The GL equations (1) and (2) can be solved numerically using the finite difference method. See [3] for details.

When the convergence of u and A_z' is achieved in all cells, the penetration depth λ and the real part of the conductivity σ_1 can be updated at the different cells of the center conductor. Thus, for each value of input current we can obtain different spatial variation of λ and σ_1 inside the center conductor of the stripline. The spatial dependence of $\sigma_1(\bar{r})$ can be calculated from $\lambda(\bar{r})$ by considering the conservation of the total number of electrons in the superconductor. The procedure described in [4] is then used to calculate the inductance L for each value of input current.

COMPARISON WITH EXPERIMENT

From the dependence of the resonance frequency on inductance, we obtain the fractional change in frequency as

$$\frac{\Delta f}{f} \approx -\frac{1}{2}\frac{\Delta L_{ac}}{L_{ac}} \tag{5}$$

Numerical results for two $YBa_2Cu_3O_{7-x}$ superconducting stripline resonators having film thickness $t = 0.3$ µm are presented. In Figure 2, the distribution of $\lambda = \lambda_L/u$ for a $YBa_2Cu_3O_{7-x}$ stripline at 77 K with $T_c = 90$ K, $\lambda_L(0) = 0.22$ µm, $\mu_0 H_c(T) = 0.1$ T and $I_{max} = 1$ A is shown. It is clear that in the ground planes and the middle region of the center strip where the current density is low, λ stays constant at its low-field value λ_L. However, near the two edges of the strip, it increases rapidly because of the high-field environment. In other words, the regions around the edges become less superconducting.

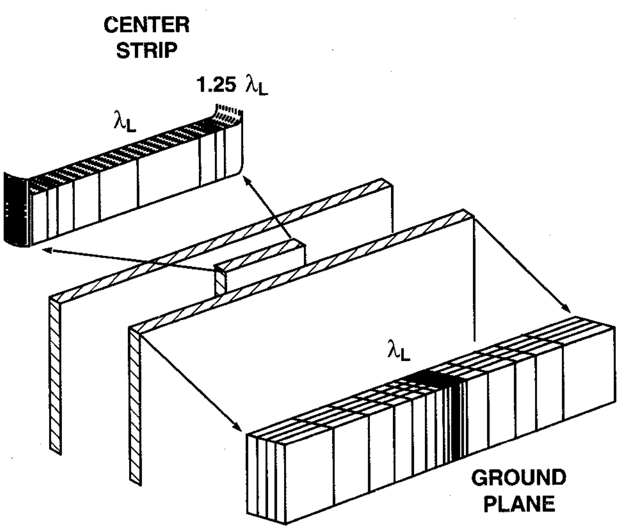

Figure 2. Distribution of the penetration depth λ in the $YBa_2Cu_3O_{7-x}$ stripline at $T = 77$ K with $T_c = 90$ K and $I_{max} = 1$ A.

Figure 3 shows the resonant frequency shift $-\Delta f/f$ versus I_{max} for the same stripline resonator. The resonant frequency shift is calculated for both $\mu_0 H_c = 0.1$ T and $\mu_0 H_c = 0.2$ T and compared with measurement. The value of Δf is measured by observing the change in frequency of the maximum of the resonance curve as a function of resonator rf current. More detail of these film and measurement has been presented in [5], in which the low-field properties as well as the high-rf-field surface resistance was presented. The fit to the shape of the curve is excellent and the value of 0.1 T for H_c is reasonable.

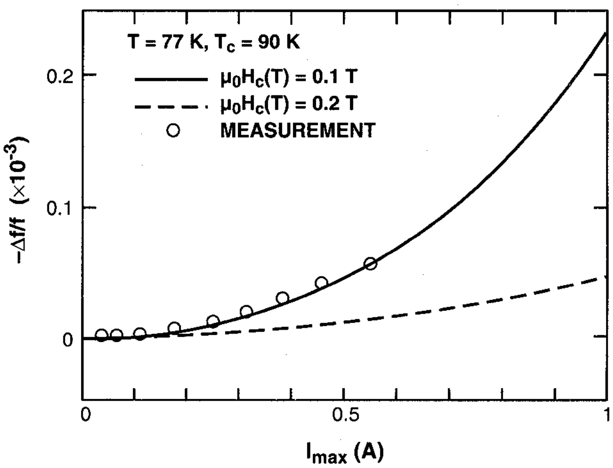

Figure 3. Fractional change in the resonant frequency of the $YBa_2Cu_3O_{7-x}$ stripline resonator at $T = 77$ K with $T_c = 90$ K and $f = 1.5$ GHz.

Figures 4 and 5 show the measurements on another $YBa_2Cu_3O_{7-x}$ specimen ($T_c = 86.4$ K and $\lambda_L(0) = 0.167$ µm) at 78 K and 75 K, respectively. In Figure 4, we can see that good fit is obtained with $\mu_0 H_c = 0.025$ T. If we assume the temperature dependence of H_c is given by $H_c(T) = H_c(0)\,[1 - (T/T_c)^2]$ we expect that $\mu_0 H_c$ at 75 K should be about 0.033 T. Again, the shape of the curve fits very well and the GL calculations reproduce the temperature variation. These results demonstrate the capability of the presented method, which is based on the GL theory, in predicting the resonant frequency shift due to the nonlinearity in superconducting stripline resonators. An extension of this work is being applied to the calculation of the nonlinear resistance R and quality factor Q of the stripline. These quantities can be obtained from the time-average power dissipation, which requires the calculation of the electric field distribution inside the superconductor. This problem is being solved in the time domain where the first London equation is used. The electric field is calculated numerically from the time derivative of both the square of the magnetic penetration depth λ and the current density $\bar{J}$. The numerical and experimental results for R(I) and Q(I) will be presented in a later publication.

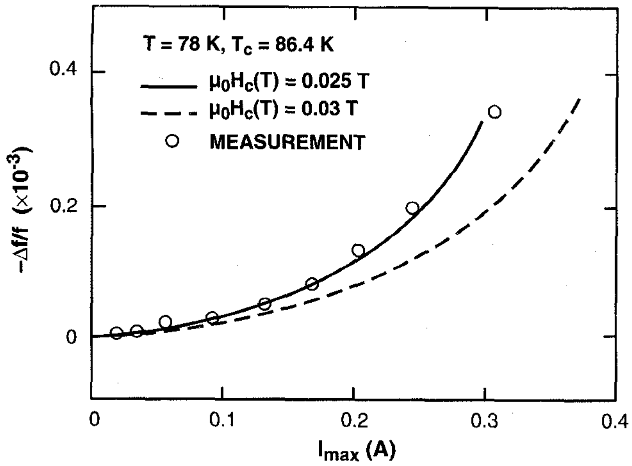

Figure 4. Fractional change in the resonant frequency of the $YBa_2Cu_3O_{7-x}$ stripline resonator at T = 78 K with T_c = 86.4 K and f = 3.0 GHz.

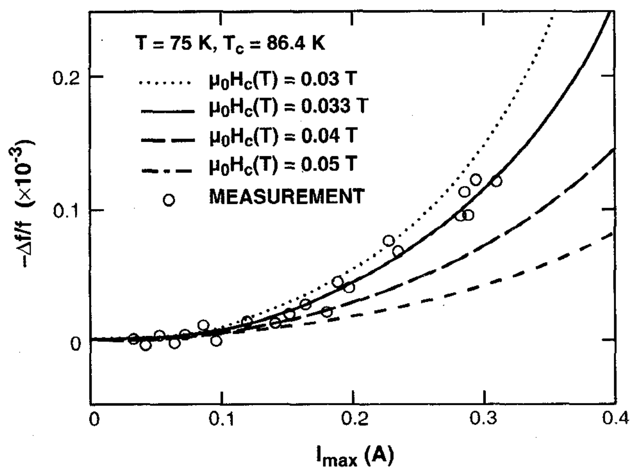

Figure 5. Fractional change in the resonator frequency of the $YBa_2Cu_3O_{7-x}$ stripline resonator at T = 75 K with T_c = 86.4 K and f = 3.0 GHz.

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